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ASTROPHYSICAL QUANTITIES OF CEPHEID VARIABLES MEASURED WITH THE NAVY PROTOTYPE OPTICAL INTERFEROMETER

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ABSTRACT

We present mean angular diameters for two Cepheid variables, α UMi and ζ Gem, determined with the Navy Prototype Optical Interferometer (NPOI). We present linear radii for these Cepheids and two additional Cepheids, δ Cep and η Aql, previously observed at the NPOI. We find the limb-darkened angular diameters of α UMi and of ζ Gem to be 3.28 ± 0.02 and 1.55 ± 0.09 mas, respectively. Using trigonometric parallaxes, we find the linear radii of α UMi, ζ Gem, δ Cep, and η Aql to be 46 ± 3, 60^{+25}_{-14}, 45^{+8}_{-6}, and 69^{+28}_{-15} $R_\odot$, respectively. We compare the pulsation periods and linear radii of this sample of Cepheids, which range in period from 3 to 11 days, to theoretical and empirical period-radius and period-radius-mass relations found in the literature. We find that the observed diameter of α UMi is in excellent agreement with the predicted diameter as determined from both surface brightness techniques and theory only if α UMi is a first-overtone pulsator.

Subject headings: Cepheids — stars: fundamental parameters — stars: individual (δ Cephei, η Aquilae, Polaris, ζ Geminorum) — techniques: interferometric

1. INTRODUCTION

Accurate stellar radii are important for the study of Cepheid mass, pulsation, and distance. Direct radius measurements of bright, nearby Cepheids allow for comparison to radii found by indirect and/or theoretical methods such as numerical models (Bono, Caputo, & Marconi 1998), the infrared flux method (Fernley, Skillen, & Jameson 1989), and surface brightness relations (Moffett & Barnes 1987; Laney & Stobie 1995). These methods are easily applied to distant Cepheids, including those in nearby galaxies (Gieren et al. 2000). Each of these indirect methods results in period-radius and period-radius-mass relations which yield different radii, and different masses, at very small and very large periods. From directly measured radii, we may make comparisons with these relations. Since there will always be Cepheids too small or too faint for direct measurement, the comparison between these indirect measurements and relations is crucial for the radius estimation of ever more distant Cepheids.

At its current magnitude limit ($m_V \sim 5$) and longest baseline (38 m), the angular diameter of four Cepheids are measurable with the Navy Prototype Optical Interferometer (NPOI): δ Cep, ζ Gem, η Aql, and α UMi (hereafter Polaris). In this paper we present mean angular diameters and compare linear radii for all four Cepheids with those in the literature and with published period-radius, period-mass-radius, and period-mass relations. Even though the sample is small, these four Cepheids span an interesting range in pulsation period and characteristics.

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2. OBSERVATIONS AND DIAMETER MEASUREMENTS

Polaris and ζ Gem were observed over the course of 2 years. Polaris was observed on 10 nights from 1997 September to November, while ζ Gem was observed for four nights: 1998 October 12, 20, and 23 and 1999 February 24. The detailed observing strategy and data reduction techniques for obtaining mean angular diameters at the NPOI are described in Nordgren et al. (1999). Armstrong et al. (2000) present the specific observations and data reduction of δ Cep and η Aql. Comparisons between the reduction method employed by Armstrong et al. (2000) and that used in this work are made at the end of this section.

Briefly, as described in Nordgren et al. (1999), squared visibilities are measured in each of 10 spectral channels, spaced evenly in wavenumber, ranging from 649 to 849 nm. A uniform-disk model is fitted to the visibility data, from which a uniform-disk diameter is derived. The uniform-disk diameters of Polaris and ζ Gem are found to be 3.14 ± 0.02 and 1.48 ± 0.08 mas, respectively. As reported in Nordgren et al. (1999), the uniform-disk diameters for δ Cep and η Aql are 1.46 ± 0.02 and 1.65 ± 0.04 mas, respectively. Figure 1 shows visibility data for the NPOI's longest baseline (east-west) for each of the four Cepheids. The data shown in Figures 1*a*–1*d* are for the night listed in each. The mean uniform-disk diameter for each is the overall mean diameter determined for that Cepheid.

Although limb darkening of evolved stars has been directly observed with the NPOI (Hajian et al. 1998), those stars are 3 times larger than the Cepheids in this study. At the spatial frequencies currently available to the NPOI, the visibility differences between limb-darkened and uniform disks for such small stars is less than the scatter in the data. Until the availability of longer baselines, limb-darkened diameters, θ_L, can be derived from uniform-disk diameters using a multiplicative conversion factor. This conversion factor is a single quadratic coefficient from Claret, Dias-Cordova, & Gimenez (1995) interpolated for the Cepheid's average specific gravity (log g) and average effective tem-

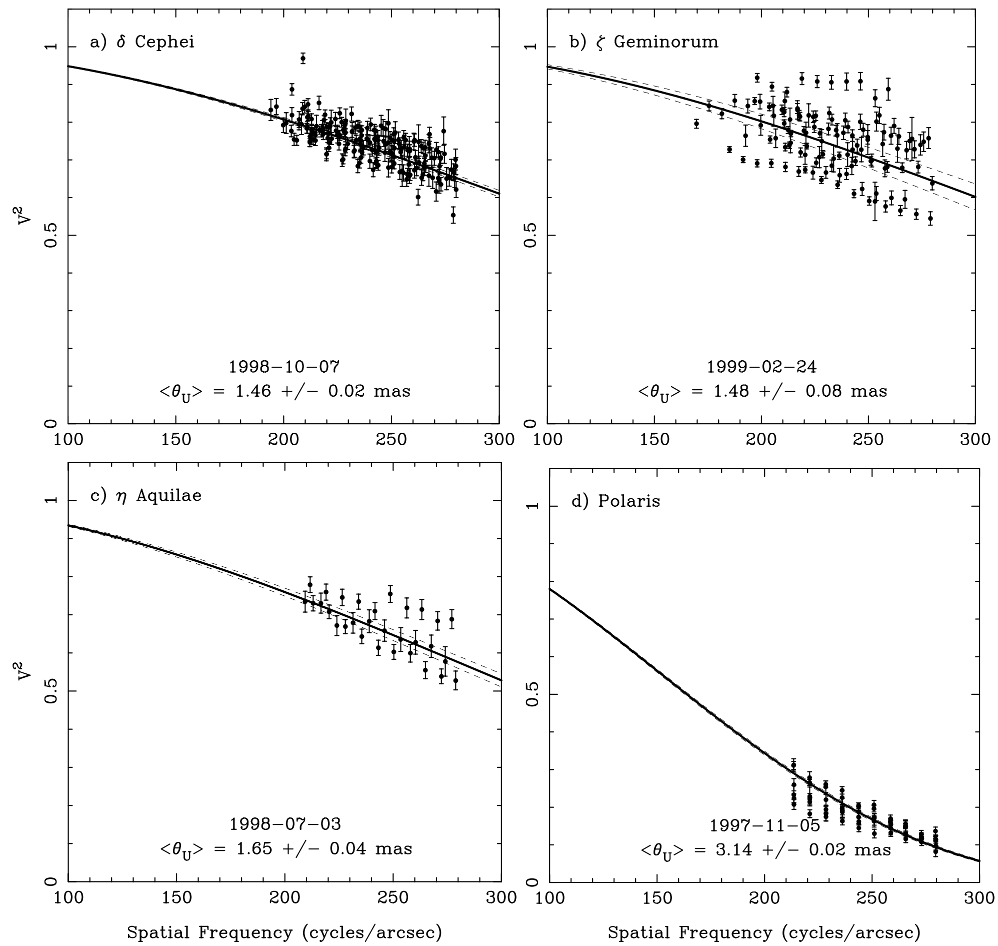

FIG. 1.—Visibility data for the four Cepheids observed with the NPOI. For each figure, visibilities for the east-west baseline are shown for a given night. The model uniform-disk diameter and error shown are the mean diameter and error of the mean for all nights that Cepheid was observed. Each figure is plotted to the same scale. (*a*) δ Cep for 1998 October 7; (*b*) ζ Gem for 1999 February 24; (*c*) η Aql for 1998 July 3; (*d*) Polaris for 1997 November 5.

perature and for the mean central wavelength of the NPOI bandwidth (740 nm). The Bright Star Catalogue (Hoffleit & Jaschek 1982) categorizes both ζ Gem and Polaris as spectral type F7 Ib. For this spectral type, Straizys & Kuriliene (1981) give a log g of 1.71 and an effective temperature of 6000 K. Using these values and the technique described in Nordgren et al. (1999), we derive a limb-darkened conversion factor (ratio of limb-darkened diameter to uniform-disk diameter) of 1.046 for both Cepheids at 740 nm. The uncertainty in this conversion factor is estimated to be on the order of 0.5% (Nordgren et al. 1999), even for ζ Gem, whose spectral type is very uncertain. With this derived limb-darkening coefficient, we find a limb-darkened diameter of 3.28 ± 0.02 mas for Polaris and 1.55 ± 0.09 mas for ζ Gem.

Using this method in Nordgren et al. (1999) resulted in a limb-darkened diameter of 1.52 ± 0.02 mas for δ Cep and 1.65 ± 0.04 mas for η Aql. Armstrong et al. (2000) use a different reduction method for the calibration of the raw visibility data for these two Cepheids (as well as two nonvariable "check stars"). In addition, limb-darkened diameters are fitted directly to the squared-visibility data without first calculating uniform-disk diameters. Armstrong et al. (2000) measure a limb-darkened angular diameter of 1.520 ± 0.014 mas for δ Cep and 1.69 ± 0.04 mas for η Aql. For the nonvariable star β Lac, Armstrong et al. (2000) derive a limb-darkened diameter of 1.909 ± 0.011 mas, while Nordgren et al. (1999) find 1.92 ± 0.02 mas. Since these two different reduction methods produced diameters equal within the errors, there is strong confidence in the robustness of the final results. The diameters for δ Cep and η Aql used throughout the rest of this work are those of Armstrong et al. (2000).

Finally, each of the four Cepheids is part of a multiple system. If the NPOI should detect light from more than one star, the visibilities measured will be depressed depending upon the position angle and separation of the system. If not taken into account, this variation will have the effect of changing the model diameter that best fits the observed data. Fortunately, each of the companions is either several magnitudes fainter than the Cepheid being observed (placing it well below the NPOI's detection threshold) or at a large enough separation ($\geq 18''$) that it is outside the NPOI's photometric field of view, or both. For example, the companion to η Aql is 4.6 mag fainter (Böhm-Vitense & Proffitt 1985) while at the same time being substantially bluer (spectral type A1 V compared to F6 Ib–G4 Ib for η Aql). Since the visibilities from only the 10 reddest channels are used to fit diameters, further chances of contamination by the companion are reduced. There is therefore no indica-

TABLE 1

CEPHEID ANGULAR DIAMETERS, DISTANCES, AND RADII

Cepheid	Period[a] (days)	θ_L[b] (mas)	π_H (mas)	π_N (mas)	d[c] (pc)	R_N ($R_\odot$)
Polaris......	3.9729	3.28 ± 0.02	7.56 ± 0.48	...	132^{+9}_{-8}	46^{+3}_{-3}
δ Cep	5.3663	1.52 ± 0.01	3.32 ± 0.58	5.0 ± 1.3	278^{+48}_{-36}	45^{+8}_{-6}
η Aql	7.1766	1.69 ± 0.04	2.78 ± 0.91	2.3 ± 1.3	382^{+150}_{-84}	69^{+28}_{-15}
ζ Gem	10.1507	1.55 ± 0.09	2.79 ± 0.81	...	358^{+147}_{-81}	60^{+25}_{-14}

[a] Period for Polaris and ζ Gem from D. Fernie 1999, private communication. Period for δ Cep and η Aql from Szabados 1980.

[b] For δ Cep and η Aql from Armstrong et al. 2000.

[c] For δ Cep and η Aql calculated from the weighted average of the *Hipparcos* and USNO parallaxes.

tion that there has been contamination of the measured diameters for any of the Cepheids due to stellar companions.

2.1. *Distances and Linear Radii*

Where there is a measured trigonometric parallax, π, the distance, d, to the Cepheid is the reciprocal of π, and the linear radius is simply $R = d \tan(\theta_L/2)$. All four Cepheids have parallaxes measured by *Hipparcos* (Perryman et al. 1997), while δ Cep and η Aql have additional parallaxes measured at the U.S. Naval Observatory Flagstaff Station (H. Harris 1999, private communication). For these two Cepheids, the distance and linear radius are derived from the weighted mean of the two measured parallaxes: 3.60 ± 0.53 mas for δ Cep and 2.62 ± 0.74 mas for η Aql. These linear radii are nearly model independent; what dependence there is enters from the conversion between uniform-disk and limb-darkened angular diameters and, as previously noted, is estimated to be at the level of $\sim 0.5\%$ of the mean radius. When the ~ 70 m baseline at the NPOI becomes operational, spatial frequencies of ~ 400 cycles arcsec^{-1} will be accessible, and at that time limb-darkened angular diameters for these stars will be able to be measured directly.

Table 1 lists the four Cepheids and includes the NPOI limb-darkened angular diameter, the *Hipparcos* parallax, π_H, USNO parallax, π_N, the distance (found from the weighted mean parallax for δ Cep and η Aql), and the NPOI's direct linear radius, R_N.

3. DIRECT RADIUS COMPARISONS

The most common method for estimating Cepheid radii and distances is the Baade-Wesselink (Wesselink 1969), or surface brightness, method (Barnes & Evans 1976). This method, of which there are several variations, relies upon observations of color and radial velocity changes (Moffett & Barnes 1987). For ζ Gem, Moffett & Barnes (1987) derive a radius of $65 \pm 12\ R_\odot$, while Scarfe (1976) derives a radius of $68 \pm 3\ R_\odot$. The average difference between these and the NPOI result (Table 1) is 10%, whereas the error of R_N toward higher values is 42%. The difference between the percent errors toward lower and higher values arises from unequal error bars for the distance and linear radius in the sixth and seventh columns of Table 1. Similarly, for δ Cep, Moffett & Barnes (1987) and Fernley et al. (1989) derive a radius of 41 ± 2 and $37 \pm 4\ R_\odot$, respectively. The average percent difference between R_N and these is 12%, which is slightly smaller than the percent error toward lower values of the NPOI radius (14%). Given the uncertainties in R_N, the measured radii for both these Cepheids are consistent with values in the literature.

Moffett & Barnes (1987) derive a radius of $55 \pm 4\ R_\odot$ for η Aql, while Fernley et al. (1989) calculate $53 \pm 5\ R_\odot$. The average percent difference with R_N is 24%, slightly larger than the percent error toward lower values of the NPOI radius (22%). Sasselov & Lester (1990), however, use high-resolution infrared spectroscopy to find a radius of $63 \pm 6\ R_\odot$ for η Aql, almost 10% larger than those found using optical spectroscopy. The photospheric lines in the high-resolution infrared spectra show asymmetries and line splitting which are interpreted to be pulsationally driven shock waves in the atmosphere. The larger radius results from the new interpretation of these spectra and projection factors derived from them specifically for the degree of limb darkening expected in the infrared. The difference between the diameter derived from the IR spectra and the NPOI diameter (Table 1) is only 9%.

Table 2 lists these previously published radii for three of the Cepheids in this paper (not including Polaris). The method which shows the least agreement with the observations reported here is the CORS method, a variation of

TABLE 2

CEPHEID RADII COMPARISON

Cepheid	R_N	R_{SB}[a]	R_{IRFM}[b]	R_{CORS}[c]	R_{IR}[d]	R_{BW}[e]
δ Cep	45^{+8}_{-6}	41 ± 2	37 ± 4	53 ± 3	...	...
η Aql	69^{+28}_{-15}	55 ± 4	53 ± 5	57 ± 3	62 ± 6	...
ζ Gem......	60^{+25}_{-14}	65 ± 12	...	86 ± 4	...	68 ± 3

NOTE.—All measurements are in units of $R_\odot$. There is no previously published estimate for the radius of Polaris.

[a] Surface brightness technique: Moffett & Barnes 1987.

[b] Infrared flux method: Fernley et al. 1989.

[c] CORS method: Ripepi et al. 1997; Caccin et al. 1981.

[d] Infrared spectroscopy and SB technique: Sasselov & Lester 1990.

[e] Optical Baade-Wesselink method: Scarfe 1976.

the surface brightness technique which is different in its mathematical computation (Ripepi et al. 1997; Caccin et al. 1981). Table 2 shows that for these three Cepheids the CORS method produces radii consistently larger than those produced by other optical surface brightness methods.

For Polaris, it has been observed that the amplitudes of the photometric and radial velocity variations have decreased steadily (Arellano Ferro 1983; Kamper, Evans, & Lyons 1984), and although there is indication that this decrease has stopped, the amplitudes of these variations are currently at the level of only 0.032 mag and ~ 1.7 km s^{-1} (Kamper & Fernie 1998). A surface brightness analysis based on such small variations is impractical: the radius change would be on the order of 0.2 $R_\odot$, representing an angular diameter change less than 0.5%. As a result, there are no published radius estimates with which we can make a comparison. For an evaluation of the accuracy of the NPOI linear radius for Polaris, we make a comparison in the following section to various published period-radius relations derived from both theory and the application of surface brightness methods to large samples of Cepheids.

4. PERIOD-RADIUS RELATIONS AND POLARIS

Once a star is identified as a Cepheid, the pulsation period is the one quantity that is always known. Period-radius relations (hereafter *P-R*) are therefore powerful tools for determining the radius of even the most distant Cepheid. Typically, *P-R* relations are of the form

$$\log R = a + b \log P , \quad (1)$$

where R is the radius in units of solar radii, P is the period in days, and a and b are determined through observation of Cepheids for which the radius can be estimated. Different methods of determining Cepheid radii have in the past tended to yield somewhat different *P-R* relations (Fernie 1984; Moffett & Barnes 1987). We present here a few representative methods from the literature. Table 3 lists the derived a and b coefficients for each method.

Bono et al. (1998) calculate theoretical *P-R* relations using full-amplitude, nonlinear, convective models for a variety of metallicities and stellar masses. The coefficients for a metallicity representing Galactic Cepheids ($Z = 0.02$) are given in Table 3.

Gieren, Moffett, & Barnes (1999) use the Baade-Wesselink (or surface brightness) technique employing V and $V-R$ photometry, with calibration by Fouqué & Gieren (1997), to derive the radii of 116 Cepheids in both the Galaxy and the Magellanic Clouds. They find no evidence for a difference between Galactic and Magellanic relations and so calculate a single relation for both. In addition, Gieren et al. (1999) find an intrinsic width to their *P-R* relation of $\log R \pm 0.03$, which allows for radii consistent with Bono et al. (1998) over the range of periods in this paper.

While still using the surface brightness technique for estimating Cepheid diameters, Laney & Stobie (1995) find that K and $J-K$ (as well as $V-K$) photometry yields more accurate results than optical photometry, due to minimal effects of gravity and microturbulence on infrared fluxes. For periods less than 11.8 days, Laney & Stobie (1995) derive smaller radii than the other two methods.

For periods less than 48 days, which is the range within which all of the Cepheids in this paper are found, the theoretical relation predicts a larger radius (~ 7%) than that found from surface brightness relations. These representative *P-R* relations are shown in Figure 2 along with the four Cepheid radii measured at the NPOI as given in Table 1. Figure 2 shows that although Polaris has the highest radius precision (owing to the most precise parallax), its radius is larger than predicted by any of the published *P-R* relations. The difference between the theoretical curve of Bono et al. (1998), the relation which predicts the largest radius, and the measured radius of Polaris is 2.6 σ given the uncertainty of only 3 $R_\odot$ in R_N. Even with the intrinsic width of the Gieren

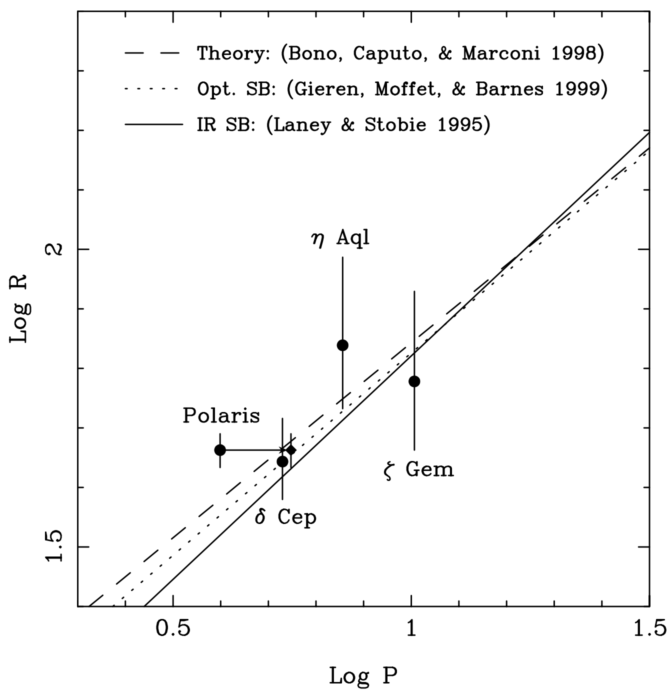

FIG. 2.—Period-radius diagram for the four observed Cepheids. Shown for comparison is the theoretical relation for Galactic Cepheids (Bono et al. 1998), the Gieren et al. (1999) optical surface brightness relation, and the IR surface brightness relation derived by Laney & Stobie (1995). The circular data point for Polaris is plotted at the observed period of 3.97 days, while the diamond is the radius of Polaris plotted at the derived fundamental period of 5.59 days.

TABLE 3

CEPHEID PERIOD-RADIUS RELATIONS

Method	a	b	$R/R_\odot$[a]	Reference
Theory	1.188 ± 0.008	0.655 ± 0.006	47.6 ± 1.0	1
Optical surface brightness......	1.146 ± 0.025	0.680 ± 0.017	45 ± 3[b]	2
IR surface brightness	1.070 ± 0.027	0.751 ± 0.026	43 ± 3	3

[a] For a period $P = 5.5957$ days, $\log P = 0.7478$.
[b] Does not include the intrinsic width of the *P-R* relation.
REFERENCES.—(1) Bono et al. 1998; (2) Gieren et al. 1999; (3) Laney & Stobie 1995.

et al. (1999) *P-R* relation, the observed radius for Polaris is too large. This problem is resolved if Polaris is a first-overtone pulsator rather than a fundamental mode pulsator. Since the ratio of the first-overtone period to the fundamental period is 0.71, an overtone Cepheid plotted on a *P-R* diagram using the log of the first-overtone period instead of the fundamental period will result in a radius larger than what the *P-R* relation would predict (Gieren, Barnes, & Moffett 1989).

The overtone nature of Polaris has been noted recently in the literature (Cox 1998; Feast & Catchpole 1997). Feast & Catchpole (1997) first used *Hipparcos* parallaxes and visual magnitudes for 220 Cepheids to calculate the Cepheid period-luminosity zero point. They find that the zero point derived from Polaris alone is brighter than that produced by the rest of the sample if Polaris is considered as a fundamental pulsator. The arrow and diamond in Figure 2 place the measured radius of Polaris relative to its fundamental period: $3.9729/0.71 = 5.5957$ days. The fourth column of Table 3 lists the radius predicted by each *P-R* relation for a $P = 5.5957$ days Cepheid. The percent differences between the observed radius of Polaris ($46 \pm 3\ R_\odot$) and the fundamental period radii predicted by Bono et al. (1998) and Gieren et al. (1999) are 3% and 2%, respectively. The excellent agreement of Polaris with these radii derived from published *P-R* relations is evidence of the overtone nature of Polaris suggested by Feast & Catchpole (1997).

5. CEPHEID MASSES AND η AQUILAE

In the same way that *P-R* relations yield radii from a known period, there are period-mass and period-radius-mass relations from which masses can be derived. The period-radius-mass relation of Fricke, Stobie, & Strittmatter (1972),

$$P = 0.025(M/M_\odot)^{-0.67}(R/R_\odot)^{1.70}\ \text{days}\ , \qquad (2)$$

when solved for mass yields

$$M/M_\odot = 4.1 \times 10^{-3}(R_N/R_\odot)^{2.54}P^{-1.49}\ . \qquad (3)$$

From this equation Gieren (1989) calculates masses for a sample of 101 Cepheids using the radii of Moffett & Barnes (1987). These masses, which we refer to as M_{GMB} in this paper, are used by Gieren (1989) to derive the radius-independent Wesselink period-mass (hereafter *P-M*) relation:

$$M_{WES}/M_\odot = 6.30 - 6.06 \log P + 6.28(\log P)^2\ . \qquad (4)$$

Using equation (3) and the radii in Table 1, we calculate masses, M_N, for the four Cepheids in our sample. These masses are presented in Table 4 with masses for the four Cepheids from Gieren (1989).

As can be seen from equation (3) and Table 4, large uncertainties in the linear radius propagate into even larger uncertainties in the mass, resulting in errors of almost 50% for δ Cep and 100% toward larger masses for η Aql. Compare this to Polaris, which has the highest radius precision and thus a mass uncertainty of only 17%. This percent error is slightly less than the 20% "accidental" error Gieren (1989) estimates for M_{GMB} based on errors in their radii of ~7%–8%.

Masses from two theoretical *P-M* relations are also listed in Table 4: evolution mass and pulsation mass (Gieren 1989). The evolution mass, M_{EV}, is calculated from stellar evolution theory,

$$M_{EV}/M_\odot = 4.90 - 1.46 \log P + 3.55(\log P)^2\ , \qquad (5)$$

whereas the pulsation mass, M_{PUL}, is calculated from period–effective temperature relations,

$$M_{PUL}/M_\odot = 5.39 - 6.08 \log P + 6.60(\log P)^2\ . \qquad (6)$$

Figure 3 shows the three *P-M* relations relative to M_N. As with the estimated error of M_{GMB}, the uncertainty in M_{EV} is estimated to be on the order of 15%–20%, while uncertainties in the effective temperature scale for Cepheids are capable of bringing the pulsation mass relation into agreement with the evolution mass (Gieren 1989). Direct diam-

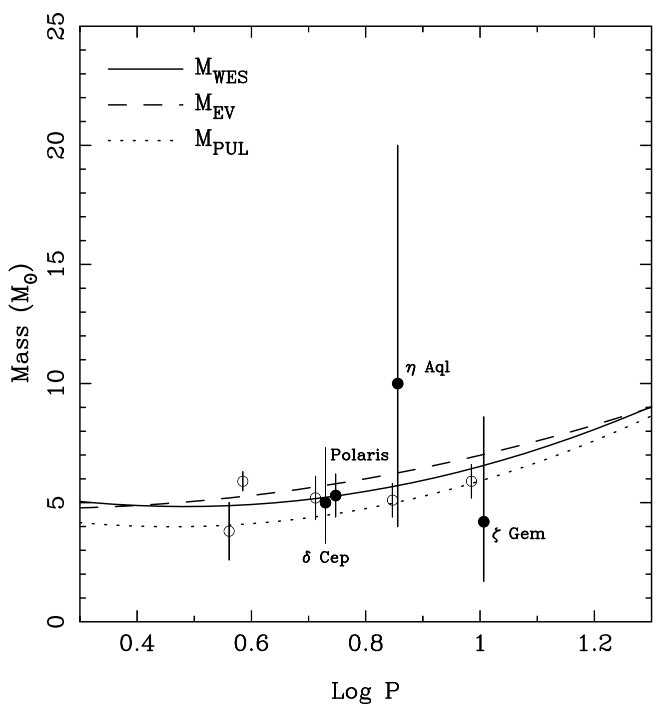

FIG. 3.—Period-mass diagram for the four Cepheids (*solid circles*). Shown are three *P-M* relations from the literature: Wesselink mass, M_{WES}, evolution mass, M_{EV}, and pulsation mass, M_{PUL}. Polaris is plotted at the fundamental period. The open circles are the binary Cepheid masses of Evans et al. (1998).

TABLE 4

CEPHEID MASSES COMPARISON

Cepheid	M_N	M_{GMB} [a]	M_{WES}	M_{EV}	M_{PUL}
Polaris[b]	$5.3^{+0.9}_{-0.9}$	...	5.3 ± 0.9	5.8 ± 0.5	4.5 ± 2.0
δ Cep	$5.0^{+2.3}_{-1.7}$	3.9	5.2 ± 0.9	5.7 ± 0.5	4.5 ± 2.0
η Aql	10^{+10}_{-6}	5.1	5.7 ± 0.9	6.2 ± 0.5	5.0 ± 2.0
ζ Gem	$4.2^{+4.4}_{-2.5}$	4.7	6.6 ± 0.9	7.0 ± 0.5	6.0 ± 2.0

[a] Gieren 1989 using surface brightness radii of Moffett & Barnes 1987.
[b] Calculated at the fundamental mode, $P = 5.5957$ days.

eter measurements will be able to address the uncertainties in this scale once one can directly measure Cepheid diameter variations and thus calculate effective temperature as a function of pulsation phase to a precision limited only by the photometry. Within the present uncertainties, then, all three *P-M* relations are consistent with each other and the NPOI masses.

The Cepheid η Aql, with a period of ~ 7 days, possesses a bump in the descending phase of its radial velocity and light curves. Radii for bump Cepheids (6 days $< P <$ 20 days) can be calculated using the bump phase and pulsation theory (Gieren et al. 1989; Fernie 1984). There is a discrepancy, however, between the radius derived in this manner and the radius (and hence mass) derived from surface brightness techniques (Gieren 1989). For a Cepheid such as η Aql, $M_{\rm GMB}$ will be larger than the predicted bump mass. Since the linear radius observed by the NPOI for η Aql is larger than that derived by Gieren et al. (1989), $M_{\rm N}$ will be larger than $M_{\rm GMB}$ and therefore in even worse agreement with the mass from the predicted bump phase method (although the error in $M_{\rm N}$ is quite large owing to the large parallax uncertainty).

Recent theoretical models of bump Cepheids by Bono, Marconi, & Stellingwerf (2000) using full amplitude, nonlinear, convective models (with no convective core overshooting) result in a Cepheid mass of $6.9 \pm 0.9\ M_{\odot}$ for a period of 11.2 days. This result agrees well with the Wesselink mass of equation (4) for an 11.2 day Cepheid which yields $M_{\rm WES} = 6.85\ M_{\odot}$. Although Bono et al. (2000) calculate the mass, and thus make their comparison to Gieren (1989) for a $P = 11.2$ day Cepheid, the implication is that the disagreement between observation and theory has been resolved in the matter of bump Cepheid masses with a resolution in favor of the larger Wesselink mass and thus closer to $M_{\rm N}$ in Table 4.

The agreement between Polaris and δ Cep and the curve for $M_{\rm WES}$ is less significant than it would at first seem since the method for calculating $M_{\rm N}$ is based upon the same theory as that used for calculating $M_{\rm WES}$. In the same way that it is desirable to compare the model-independent Cepheid radii of Table 1 to radii derived through indirect methods, it is desirable to do this for mass. A number of Cepheids are located in binary systems (including all four Cepheids in this study, as noted earlier). Already, Evans et al. (1998) have used spectroscopy to calculate the masses of five Cepheids in binaries: U Aql, S Mus, V350 Sgr, Y Car, and SU Cyg. Using ground-based optical and satellite ultraviolet spectra, the mass ratio of the two members of the binary were found. Inferring the mass of the companion based on the spectral type yielded the mass of the Cepheid. These five Cepheids are plotted in Figure 3 as open circles and show good agreement with the *P-M* relations and the NPOI observations. With long enough baselines, optical interferometry will be able to image the orbits of Cepheids as has already been done for binaries of non-Cepheids (Benson et al. 1997; Hummel et al. 1998). In conjunction with radial velocities from spectroscopy, all the orbital elements of the system, including the mass and distance, will be directly determined and independent of all models.

6. FUTURE OBSERVATIONS

At the present time, only four Cepheids have had their diameters measured with the NPOI. Over the next 2 years, as the longest baseline available increases from 38 to 440 m, the number of Cepheids resolvable and the precision of their measurements will increase by at least a factor of 5. Figure 2 shows that at the present the measured linear radii are consistent with each of the published *P-R* relations. With the increased precision of the angular diameter measurements and the increased precision of parallax observations (also undertaken at the USNO), it will be possible to differentiate between the various *P-R* relations which are seen to diverge in Figure 2 for periods shorter than 30 days. This is precisely the range of periods in which Cepheids observable by the NPOI are located.

7. SUMMARY

Optical long-baseline interferometry has successfully measured the mean angular diameters of the four brightest Cepheid variables in the northern sky. These angular diameters coupled with trigonometric parallaxes have produced virtually model-independent linear radii. These radii are compared to radii in the literature which have been derived from a variety of Baade-Wesselink, or surface brightness, methods. The agreement between the direct radius determinations presented here and published indirect radius estimates is quite good. The differences are $\sim 10\%$, better than the error in $R_{\rm N}$, which is on the order of 20%–40%. For η Aql, the derived linear radius is in marginal agreement with the optical surface brightness results, but it is in very good agreement with the radius estimated from infrared spectroscopy by Sasselov & Lester (1990). For Polaris, the radius precision (6%) is high enough that we are able to confirm its overtone nature. At a period of 3.97 days, a radius of $46 \pm 3\ R_{\odot}$ is inconsistent with the published *P-R* relations of Bono et al. (1998) and Gieren et al. (1999). Only as an overtone pulsator with a fundamental period of 5.59 days is Polaris in agreement with these *P-R* relations, confirming the findings of Feast & Catchpole (1997) using completely independent means. At a period of 5.59 days, the Wesselink mass of Polaris is found to be in excellent agreement with the period-mass relation of Gieren (1989).

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